Tutorial Note for Math2012E

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1 vector

- representation (coordinate)
- calculation/application
  - addition
  - scalar multiplication
  - dot product
    - $$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos\theta = x_1x_2 + y_1y_2 + z_1z_2$$
    - linear to addition/scalar multiplication
    - commutative
  - cross product
    - $$\vec{u} \times \vec{v} = |\vec{u}||\vec{v}| \sin\theta\vec{n} = \begin{vmatrix} i & j & k \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$$
      and $\vec{n}$ is determined by right hand law.
    - linear to addition/scalar multiplication
    - anti-commutative
    - $\vec{u} \times (\vec{v} \times \vec{w}) = (\vec{u} \cdot \vec{w})\vec{v} - (\vec{u} \cdot \vec{v})\vec{w}$
    - $Volume(\vec{u}, \vec{v}, \vec{w}) = |\vec{u} \cdot (\vec{v} \times \vec{w})| = \left|\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}\right|$

2 line

- equation
  - in vector
    $$\vec{r}(t) = \vec{r_0} + t\vec{v}$$

  - In coordinate,

$$x = x_0 + tv_1$$
$$y = y_0 + tv_2$$
$$z = z_0 + tv_3$$

  - intersection of two planes cf. section 3: plane

- distance of point and line

$$d(Q, l) = \frac{|\vec{PQ} \times \vec{l}|}{|\vec{l}|}$$

3 plane

- equation
  - vector equation $\vec{n} \cdot \vec{PQ} = 0, P$ fixed point
  - in coordinate $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$
  - simplified $Ax + By + Cz = D$, where $D = Ax_0 + By_0 + Cz_0$
- intersection of two planes, cf. section 2: equation of line

$$\vec{v} = \vec{n_1} \times \vec{n_2}$$

- distance of point and plane

$$d(Q, \alpha) = \frac{|\vec{PQ} \cdot \vec{n}|}{|\vec{n}|}, P \in \alpha$$

- angle between two intersecting planes

$$\cos\theta = \frac{|\vec{n_1} \cdot \vec{n_2}|}{|\vec{n_1}||\vec{n_2}|}, \theta \in [0, \frac{\pi}{2}]$$